Lecture 12

$\boxed{4.}$ $Q(x)$ has repeated irreducible quadratic factors.
For every factor of the form, $(a_i x^2 + b_i x + c_i)^k$, $k > 1$, we include the terms

$$\frac{A_1 x + B_1}{a_i x^2 + b_i x + c_i} + \cdots + \frac{A_k x + B_k}{(a_i x^2 + b_i x + c_i)^k}$$

For example, the decomposition of:

$$\frac{6x^4 - 2x + 1}{x^5 + 3x^4 + 3x^3 + x^2} =$$

Ex: Compute $\displaystyle\int \frac{dx}{x(x^2+4)^2}$

If the numerator is a polynomial of higher degree than the denominator, then we must do long division first to write

$$\frac{P(x)}{Q(x)} = S(x) + \frac{R(x)}{Q(x)}$$

so $\deg(R) < \deg(Q)$

<u>Ex</u>: Compute $\displaystyle\int \frac{x^3-4x-10}{x^2-x-6}\,dx$

Sometimes it may be necessary to make a substitution before attempting partial fractions.

Ex: Compute $\displaystyle\int \frac{e^{2x}}{e^{2x}+3e^{x}+2}\,dx$

<u>Ex</u>: Evaluate $\int \frac{dx}{x^2 + x\sqrt{x}}$